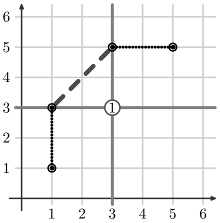
Statement

Problem J: Transport Pluses

In this problem, we have to get from point A to point B on a plane.

- You can go from one point to another directly, paying for the Euclidean distance between them.
- Also, you can use $n \leqslant 100$ transport pluses (x_i, y_i).
- Pay t, and get from any point with $x = x_i$ OR $y = y_i$ to any other such point.
- The goal is to get from A to B with minimum possible total cost.

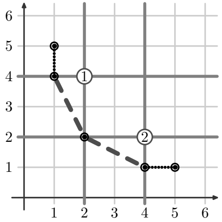


What Can a Path Look Like

Consider a travel path from A to B: some direct travel, using a plus, another direct travel, using another plus, and so on.

- Each pair of pluses is connected.
- So, if we use at least two pluses, our travel can skip every step between the first and the last plus.
- To move from a point to plus i (and the other way too), we move vertically to its x_i or horizontally to its y_i, whichever is closer.

This leaves only a small number of cases to consider as possible optimal solutions.



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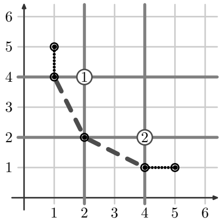
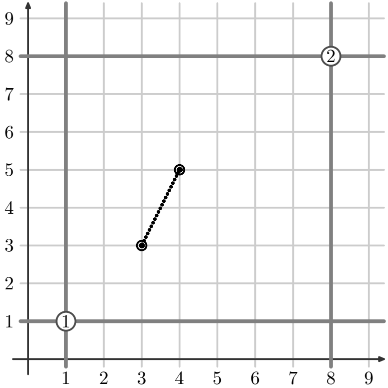


Possible Cases

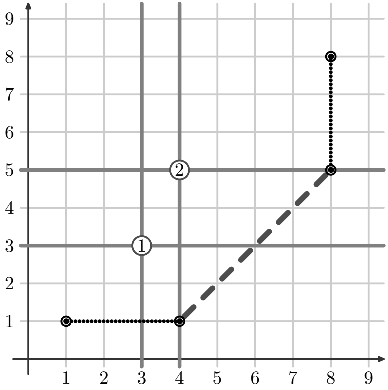
Case 1: direct travel.



Can check in $\mathcal{O}(1)$.

Possible Cases

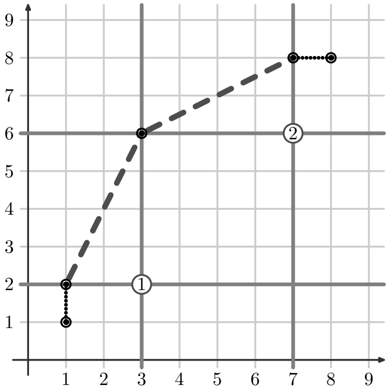
Case 2: travel to a plus, use it, travel to destination.



Can check in $\mathcal{O}(n)$.

Possible Cases

Case 3: travel to a plus, transfer to another plus, get closer to destination, go directly.



Can check in $\mathcal{O}(n^2)$, or in $\mathcal{O}(n)$ if we precalculate distances from A and B to each plus.

Statement

Problem L: "Memo" Game With a Hint

- In this problem, we play the "Memo" game:
  - There are 50 cards lying face down.
  - On the faces, there are 25 pictures, each appears twice.
  - In a turn, open one card, then open another, trying to find the same picture.
  - If the pictures are the same, leave the pair face up.
  - If the pictures are different, turn the two cards face down again.
  - The goal is to open all cards in as little turns as possible.

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  - If the pictures are different, turn the two cards face down again.
  - The goal is to open all cards in as little turns as possible.
- During the preparation phase, we can cheat: look at all the pictures, and then rotate some of the cards 180 degrees before turning them face down.
- In the main phase, we don't remember the pictures, but we see the rotations.

Basic Strategy

Here is the strategy without hints.
Before each turn, there are the following types of cards:

- known pairs face up,
- known pairs face down,
- known cards without a known pair,
- unknown cards.

On each turn:

- If there is a known pair face down, open it.
- Otherwise, open an unknown card.
- If we know its pair, open the pair.
- Otherwise, open another unknown card.
- Either we found a new pair by luck, or we now know 2 more cards.

This strategy has ≈ 14.83 misses per game. We want 13.50 misses per game on average. There are several ways to get there.

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Using the Hint: Solution 1

Here is a simple idea for the solution.

- Note that, in 13 turns, we can learn about 26 cards.
- In preparation phase, for each of the 25 pairs, rotate one card and don't rotate another.
- In playing phase, spend the first 13 turns (and 13 misses) to see all rotated cards.
- In the next turns, open a non-rotated card. We already know where its pair is.
- With a patch, can do 12 misses instead of 13.

Using the Hint: Solution 2

Here is a mechanical solution.

- Treat the hint as 50 bits of information.
- Write down the information allowing to open pairs as long as we have bits for it.

For example:

- Write down the number of pair for the leftmost card (49 choices).
- Out of the 48 remaining cards, pick the leftmost one and write down the number of its pair (47 choices).
- And so on.
- We can transfer information for 9 turns: $49 \cdot 47 \cdot 45 \cdot 43 \cdot 41 \cdot 39 \cdot 37 \cdot 35 \cdot 33 = 304\,513\,870\,485\,825$, and it is less than $2^{50} = 1\,125\,899\,906\,842\,624$.
- After that, use the basic strategy, but for $16 \cdot 2 = 32$ remaining pairs.
- The number of misses is ≈ 9.31. We can use the gap to skip some technicalities.

Questions?