# Sweet Remainders!

| | |
|---|---|
| Input file: | *standard input* |
| Output file: | *standard output* |
| Time limit: | 2 seconds |
| Memory limit: | 1024 mebibytes |

Given are integers $n, m > 0$, as well as integers $a_1, a_2, \ldots, a_n \in \{-1, 0, 1, \ldots, m-1\}$. Construct a tree $T$ with $n$ vertices $V = \{v_1, v_2, \ldots, v_n\}$ with the following condition: for each vertex $v_i$ where $a_i \neq -1$, the remainder of the degree $\deg_T(v_i)$ when divided by $m$ must equal $a_i$. If $a_i = -1$, then there are no restrictions on the degree of $v_i$.

## Input

The first line of input contains an integer $t$, the number of test cases ($1 \le t \le 5 \cdot 10^5$). For each test case:

The first line contains two integers, $n$ and $m$, denoting the number of vertices in the tree and the modulus ($1 \le n \le 5 \cdot 10^5$; $1 \le m \le 2 \cdot 10^9$).

The next line contains $n$ integers $a_1, a_2, \ldots, a_n$ denoting the required remainders ($-1 \le a_i < m$).

The sum of $n$ across all test cases does not exceed $5 \cdot 10^5$.

## Output

For each test case, if such a tree with $n$ vertices exists, output "`YES`" (in any case), followed by $n-1$ lines with two integers each denoting the endpoints of each edge. Otherwise, output "`NO`" (in any case).

## Example

| *standard input* | *standard output* |
|---|---|
| 6<br>1 5<br>0<br>2 3<br>1 1<br>5 4<br>0 1 1 1 1<br>6 2<br>1 0 0 0 0 1<br>3 2<br>1 1 1<br>4 10<br>3 3 1 1 | YES<br>YES<br>1 2<br>YES<br>1 2<br>1 3<br>1 4<br>1 5<br>YES<br>1 2<br>2 3<br>3 4<br>4 5<br>5 6<br>NO<br>NO |

## Note

In the first test case, we are asked to build a tree with $n = 1$ vertex, $m = 5$, and degree residue array $b = [0]$. With only one vertex, there can be no edges, so there is exactly one possible tree: a single vertex of degree 0. Since $0 \bmod 5 = 0$, it satisfies the requirement; hence the answer is "`YES`".

In the second test case, we need a tree on $n = 2$ vertices with $m = 3$ and residues $b = [1, 1]$, meaning both degrees must be congruent to 1 modulo 3. On two vertices, there is only one tree: the single edge $1-2$. Its degrees are $(1, 1)$ and indeed $1 \bmod 3 = 1$, so the answer is "`YES`", and printing "`1 2`" is valid.

In the third test case, we need $n = 5$, $m = 4$, with residues $b = [0, 1, 1, 1, 1]$: the first vertex must have degree divisible by 4, while the other four vertices must have degree $\equiv 1 \pmod 4$. A natural construction is a star centered at vertex 1: connect 1 to every other vertex. Then $\deg(1) = 4$ and $4 \bmod 4 = 0$, while each of $2, 3, 4, 5$ has degree 1 and $1 \bmod 4 = 1$. Therefore, the answer is "`YES`", and edges "`1 2`", "`1 3`", "`1 4`", "`1 5`" work.

In the fourth test case, we need a tree with $n = 6$, $m = 2$, residues $b = [1, 0, 0, 0, 0, 1]$, i.e. the endpoints must have odd degree and the four middle vertices must have even degree. Modulo 2, this is purely a parity constraint. The simple path $1-2-3-4-5-6$ fits perfectly: vertices 1 and 6 have degree 1 (odd), and vertices $2, 3, 4, 5$ have degree 2 (even). Hence the answer is "`YES`", and the path edges are correct.

In the fifth test case, we are asked to build a graph (and thus, in particular, a tree) on $n = 3$ with $m = 2$ and $b = [1, 1, 1]$, so every vertex must have odd degree. This is impossible: in every undirected graph, the number of odd-degree vertices is always even, so having three odd degrees cannot happen for any set of edges. Therefore, the verdict is "`NO`".

In the sixth test case, we have $n = 4$, $m = 10$, $b = [3, 3, 1, 1]$. Since $m > n - 1 = 3$, the residue modulo 10 equals the degree itself, so we require exactly $\deg = [3, 3, 1, 1]$. That forces two vertices to be adjacent to all others (degree 3 in a 4-vertex graph), and when $n > 2$, any such pair necessarily forms a cycle together with any third vertex. A tree cannot contain cycles, so the verdict is also "`NO`".