1. Problem Name

Binary Fixed-Coefficient Matrix Multiplication Optimization

2. Background

Matrix multiplication is widely used in industries such as AI computing, communications, and storage. For example, matrix multiplication is involved in forward propagation and back propagation in neural networks, filtering in signal processing, and decoding and error correction in communication and storage protocols. Therefore, the optimization of the time complexity and space complexity of matrix multiplication will affect the performance and power consumption of the related product. In industry, a typical case is multiplication of a constant matrix and a variable matrix or a variable vector. This competition considers binary matrix multiplication, a special constant coefficient matrix multiplication, and the specific matrix derived from a classical algebraic error correction code called Reed-Solomon (RS) code.

In RS code, the parity check matrix is designed according to the characteristics of polynomials over finite fields with the best tradeoff between information rate and error-correcting performance. RS codeword decoding involves matrix-vector multiplication in the finite field $\mathbb{F}_{1024}$ and can be represented as a binary matrix multiplied by a binary vector. Our objective is to decompose the binary matrix to reduce calculation overhead as much as possible.

Consider any element of the finite field $\mathbb{F}_{1024}$, which can be expressed as a polynomial of degree 10 or less with coefficients in the elements of $\mathbb{F}_2$. The addition and multiplication of the elements are the corresponding polynomial addition and multiplication modulo irreducible polynomial $p(x) = x^{10} + x^3 + 1$. If $\mathbb{F}_{1024}$ is regarded as a 10-dimensional vector space over $\mathbb{F}_2$ with bases $\{1, x, x^2, \ldots, x^9\}$, since any element m, a, b in $\mathbb{F}_{1024}$ satisfies $m * (a + b) = m * a + m * b$, the multiplication by m is a linear map from the vector space $\mathbb{F}_2^{10}$ to itself, which can be represented by a 10×10 matrix whose values are from $\mathbb{F}_2$. In this way, the matrix-by-vector computation over the finite field $\mathbb{F}_{1024}$ becomes a larger binary matrix multiplied by a binary vector. For example, consider the matrix $M \in \mathbb{F}_{1024}^{32\times30}$ multiplied by the vector $v \in \mathbb{F}_{1024}^{30}$, which after transformation is the matrix $M' \in \mathbb{F}_2^{320\times300}$ multiplied by the vector $v' \in \mathbb{F}_2^{300}$.

3. Problem description

We call a matrix a "binary matrix" if and only if its elements are 0 or 1. Matrix M is a binary matrix of size $n \times m$ that can be decomposed into the product of at most $K \le K_{max}$ matrices, $M = M_1M_2\ldots M_K$. (The operations are performed on $\mathbb{F}_2$, i.e., for integers a_k, b_k we have $\sum_k a_kb_k = \sum_k a_kb_k \mod 2$). The k-th matrix M_k is of size $n_k \times m_k$, satisfying $n_1 = n, m_K = m$ and for all $1 \le k < K, m_k = n_{k+1} \le n_{max}$. The size of the decomposed matrix and the number of 1's are positively correlated with the cost of implementing matrix multiplication. The cost function is defined as

$$\mathrm{cost}(\{M_k\}_1^K) = \sum_{k=1}^{K} |M_k| - n_k \ ,$$

where $|M_k|$ denotes the number of 1's in the matrix M_k. Your goal is to minimize the cost.

4. Problem Objectives

1. Ensure that the number of $\{M_k\}$ matrices and the size of each matrix meets the requirements, no matrix has a row containing all 0's, and the product of the matrices is equal to M.
2. The objective is to minimize the cost function.

5. Scoring

The score of a test case is (cost(the input matrix undecomposed) – cost(the output)) / cost(the input matrix undecomposed)*100. If the output of a test case is invalid, the score will be zero. Your submission will be scored on a total of 20 test cases by summing up the score of each case. We will provide you with 10 test cases. During the contest, you will receive feedback on the 3 of the provided test cases. Only your last submission will be evaluated on all 20 test cases, and the results will be announced at the awards ceremony.

6. Test cases

Input:

The first line contains two integers $0 < n \leq 300, 0 < m \leq 200$.

The second line contains two integers $K_{max} \leq 20, n_{max} \leq 2000$, indicating the maximum number of output matrices and the maximum number of rows and columns of each matrix.

The next n lines contain m integers indicating the elements in the matrix M. It is guaranteed:

(1) every element of M is either 0 or 1.

(2) each row of M has at least one 1.

Output:

The first line contains an integer K, indicating the number of decomposed matrices. The output of the K matrices follows in K groups.

For each group, output a line containing two positive integers n_k, m_k, indicating the size of matrix M_k.

Then, output n_k lines, each containing m_k integers which are either 0 or 1, indicating the elements in matrix M_k. Each row of every matrix must have at least one element equal to 1.

Input sample	Output sample
4 4 20 20 1 1 0 0 0 1 1 1 0 0 1 1 1 1 0 1	2 4 4 1 0 0 0 0 1 1 0 0 0 1 0 1 0 0 1 4 4 1 1 0 0 0 1 0 0 0 0 1 1 0 0 0 1

In this sample, the input matrix is $M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{pmatrix}$. The output sample represents the decomposition of the matrix M into

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

In this sample, the cost is $(6 - 4) + (6 - 4) = 4$. If the input matrix is output directly, the cost is $10 - 4 = 6$.